In industries like health care, consumer goods and agriculture, shortages are widely observed and the consequences can be costly. One of the main drivers of such shortages is the uncertain nature of supply and demand. To reduce uncertainties, sufficient information about supply and demand can be obtained by gathering relevant data (e.g., auditing suppliers and conducting market research). In this paper, we conduct an analysis to examine the impacts of supply uncertainty, demand uncertainty and uncertainty reduction efforts on production quantity and total cost. We show that in the absence of uncertainty reduction efforts, when the financial consequences of shortages are large or when the unit benefit is large, supply uncertainty is more costly than demand uncertainty. In addition, exerting supply uncertainty reduction effort always causes the firm to produce fewer units than exerting demand uncertainty reduction effort. Although supply uncertainty reduction effort delivers a larger degree of improvement to total cost (and hence, is more efficient), reduced supply uncertainty still leads to a higher system cost than does reduced demand uncertainty.

Key words: Demand Uncertainty; Newsvendor Model; Supply Uncertainty; Uncertainty Reduction Effort

1. Introduction

Shortages and their related consequences are widely observed in the healthcare industry. According to Clapp et al. (2013), over 300 drugs experienced shortage issues as of July 2013, up from 211 just one year earlier. These shortages are costly to healthcare providers, patients and governments alike (Federgruen 2012). To illustrate, CBC News (2011) reported that shortages in chemotherapy drugs caused a sharp rise in prices, putting both patients and doctors in difficult situations. In response to the public’s demand to know why these shortages keep happening, Canada’s federal government assigned a high priority to the task of finding a remedy for drug shortages (Health Canada 2013, Duffin 2014).

Uncertainty in supply represents an important driver of drug shortages. For example, the vaccine-manufacturing process involves the use of chicken eggs and is therefore subject
to uncertain yields due to the manufacturing process itself and the quality of the eggs (e.g., Deo and Corbett 2009, Arifoglu et al. 2012). Furthermore, the American Society of Health-System Pharmacists indicates that cancer drug shortages occur due to supply issues, such as problems with the quality of the ingredients (ASHP 2013). Uncertainty in demand represents another equally important driver of drug shortages. For example, Tamiflu, an antiviral flu drug, was stocked out in 2013, mainly because demand unexpectedly doubled from the previous year (The Canadian Press 2013). Demand for flu shots is also difficult to predict, even for an average flu season, leaving officials wondering, “What makes Canadians clamor for flu shots one year and shun them the next?” (Grant 2014). A recent news article shows that rather than improving, the issue of drug shortages is actually getting worse (Favaro and St. Philip 2016).

Besides being piqued by the headlines about drug shortages, our interest in this topic started with discussions with two contacts in the high tech industry, where knowing which uncertainty is costlier and reducing uncertainty in both demand and supply is crucial. (The names of our interview subjects have been hidden for confidentiality reasons.) Our first industry contact was Intel’s global commodity manager. During our discussion, the interviewee stated that “managing supply and demand in our business is extremely challenging due to the manufacturing yield uncertainty and capacity adjustments [of expensive equipment] and the quick and unpredictable changes in demand [due to high turnover in technology products]. There are ways to reduce uncertainties in manufacturing [e.g., process improvements] and demand [e.g., conducting market research], but these are expensive undertakings, and we need to know the effect and benefit of the improvement before undertaking such efforts. Reduction in uncertainties has a positive direct effect on our performance.”

Our second industry contact was the logistics planning director for one of the largest consumer electronics companies in Korea. During our interview, the director elaborated on shortages: “Shortages are an ongoing problem for our company. In the consumer electronics industry, product life cycles are getting shorter and shorter, which has an impact on both
supply and demand. Let’s start with the supply side. We are required to manufacture new products with brand new specifications. This adds uncertainty to production yield. Furthermore, new suppliers are often used to support new products launches, and we are not usually fully informed as to the new suppliers’ performances, thus adding more uncertainty to the supply side. It is not uncommon that a supplier’s deliverable does not meet our standards, and we discard its delivery and end up with fewer items than we originally planned to deliver. Short product cycles also affect the demand side and make demand forecasting more difficult, as we need to forecast for new products that were not in the market before. Heavy competition in the high tech industry adds another layer of difficulty in this forecasting. We mostly end up with stockouts in some markets for our new products.” Our interview subject continued: “Ideally, we would like to reduce both supply uncertainty and demand uncertainty. However, resources are limited and, more importantly, we would like to know which one to go after first - that is, which one is more profitable. Based on our experience, we think supply uncertainty is more costly than demand uncertainty for our firm. However, it would be nice to confirm this point with research-based evidence.”

Based on these interviews, we aim to quantify the effect of uncertainty reduction for both supply and demand, and to determine which type of uncertainty reduction is more beneficial.

There are numerous examples of shortages in industries other than health care. In the agriculture industry, cases often arise wherein a critical chemical ingredient needed by farmers is out of stock due to an unforeseen increase in seeding (demand) and insufficient inventory of the chemical (supply) (Prendergast 2013). In the retail industry, Canadians were initially excited about Target’s expansion into Canada, but their retail enthusiasm resulted in severe stockouts (Strauss 2013) and, ultimately, a complete shutdown of all 133 Target stores in Canada (Evans 2015). Further, many consumers regularly wait weeks for the latest iPhone because of ongoing high demand (Jordan 2012). Because of its commonality across a wide range of industries, the topic of shortages has always been of interest
to practitioners and academics.

In the classic newsvendor (NV) problem, supply is assumed to be deterministic, and therefore, shortages are solely caused by an uncertainty in demand. However, as described in the previous paragraphs, shortages may also be attributed to an uncertainty in supply. To deal with shortages and reduce uncertainties in demand and supply, firms can exert costly efforts to obtain data that will help them gain a better understanding of the subject. For example, a firm can audit its raw material providers to reduce the uncertainty in its production process or to improve its estimation of the production process (e.g., on-time delivery and quantity delivered), such that the firm can obtain the supply information with lower uncertainty (Kitamura et al. 2010, Yang et al. 2012). Alternatively, a firm can conduct market research and survey potential users to improve demand forecast (Hess and Lucas 2004).

In this paper, we examine a model where there is a newsvendor that needs to decide the order quantity in the presence of demand and/or supply uncertainty. This firm can exert effort to reduce uncertainty. To obtain analytical insights, we first consider each uncertainty separately. Then we verify the robustness of our result by considering a system where both types of uncertainty exist simultaneously. In particular, this paper addresses the following research questions:

1. What is the profitability of each uncertainty reduction effort?

2. What are the impacts of uncertainty reduction efforts on production quantity and total cost?

We find that, in the absence of effort, if the financial consequences of shortages are substantially larger than the unit production cost, then supply uncertainty is more costly than demand uncertainty. However, when the firm has the option of exerting uncertainty reduction efforts, we find that supply uncertainty reduction effort is more profitable than demand uncertainty reduction effort. Consequently, the optimal effort exerted to reduce supply uncertainty is larger than the effort exerted to reduce demand uncertainty. Yet even though supply uncertainty decreases as more efforts are exerted, it still results in a higher
total cost when compared with demand uncertainty. Moreover, as a result of these efforts, we find that demand uncertainty would always cause the firm to produce more than would supply uncertainty.

Our results imply that a firm facing both supply and demand uncertainty should first focus efforts to reduce supply uncertainty (e.g., establishing better communication with its suppliers, finding and working with more suppliers and improving production yield) if shortage cost is relatively large, and the firm should exert effort to decrease demand uncertainty (e.g., market research for its products, targeted consumer surveys and better demand forecasting) first if shortage cost is relatively small.

In the next section, we present a review of the relevant literature. In Section 3, we present our framework for the no-effort case, whereas the analysis with the efforts is provided in Section 4. We conclude the paper in Section 5, and our proofs are provided in the Appendix.

2. Literature Review

The main contribution of our study is to provide an analysis of uncertainty reduction (with respect to both demand and supply), offering a comparison with and without efforts. Therefore, our paper relates to the literature on supply uncertainty and demand uncertainty, and on how a firm can exert effort to minimize these uncertainties.

The first block of literature addresses the topic of supply uncertainty. Cho (2010) investigates a problem with composition selection for an influenza vaccine. The timing of the composition selection matters since there is a tradeoff between yield rate and effectiveness of the vaccine. Deo and Corbett (2009) combine competition with supply uncertainty and find that, up to a certain threshold of supply uncertainty, entry into the market becomes more attractive compared to no supply uncertainty. Arifoglu et al. (2012) develop an epidemiological and economical model to deal with random production supply and rational customer behavior. Arifoglu (2012) extends Arifoglu et al. (2012) by imposing mechanisms
Supply and Demand Uncertainty Reduction Efforts and Cost Comparison

(e.g., taxes and subsidies) for rational customers and by comparing their model’s performance with the outcomes of other models from the literature. Ozaltin et al. (2011) model supply uncertainty to optimize the social benefits of annual influenza vaccination.

Within the literature on supply uncertainty, some papers study supply chain coordination. Chick et al. (2008) perform disease modeling and study random supply with a newsvendor model with no information asymmetry. Chick et al. (2012) extend Chick et al. (2008) by considering private information on supply and the option of late delivery, where the buyer’s objective is to design a contract to minimize information rent due to private information on random supply of the supplier. Yang et al. (2009) examine supply disruptions with a manufacturer and a supplier under asymmetric information, where the supplier has private information about supply disruptions and pays a penalty if shortages occur. In a follow-up paper, Yang et al. (2012) extend their study to a system with two suppliers to quantify supplier reliability competition and diversification.

The second block of literature relates to studies of demand uncertainty. Numerous papers use the newsvendor model (e.g., Petruzzi and Dada 1999, Khouja 1999, Van Mieghem and Rudi 2002, Boyacı and Özer 2010). Other areas that deal with uncertain demand include supply chain management (e.g., Lariviere 1999, Bernstein and Federgruen 2005, Kaya and Özer 2011, Yang et al. 2014) and marketing (e.g., Raju and Roy 2000, Gal-Or et al. 2008). Tang et al. (2012) consider a newsvendor that faces random demand and random yield. The pricing decisions are made after observing the random supply, but before knowing the random demand. They consider both fixed and dynamic pricing policies, and find that dynamic pricing has a more significant impact when the uncertainty in demand is small. Marschak et al. (2015) confirm that a firm can gather information to reduce uncertainty. The information gatherer provides the demand distribution, which the firm uses to calculate the optimal order quantity. The literature also suggests different approaches to solve the demand uncertainty with information gathering, such as market forecast analytics (Glock and Ries 2012), chance constrained formulation (Abad 2014), utility based (Sayın
et al. 2014), minimax regret (Wang et al. 2014) and elastic p-Robustness (Jabbarzadeh et al. 2015).

The next block of literature consists of papers that consider both supply and demand uncertainties simultaneously. The papers in this segment do not consider any effort exertion to reduce uncertainty. Wang and Gerchak (1996) develop a stochastic dynamic program for a production planning problem with random yield and random demand, and characterize the optimal policy. Outsourcing and supply chain decisions in the presence of supply and demand uncertainty are studied in Kouvelis and Milner (2002). The authors determine optimal capacity investment decisions and investigate how uncertainties affect these decisions. Li and Zheng (2006) examine random supply and random demand for inventory replenishment and pricing, using single-item, periodic-review and price-sensitive period demands. Schmitt et al. (2010) study an inventory system with random supply and demand. They model supply disruptions, random demand and uncertain yield simultaneously. Federgruen and Yang (2008) consider a supplier selection problem where the buyer selects the supplier based on the shortfall probability. These authors develop a planning model for a decision-maker who faces uncertain demand for a single item that can be obtained from multiple suppliers whose yields are random. Sting and Huchzermeier (2014) model correlated but random demand and supply uncertainty for operational hedging and diversification for a single firm’s production and investment decisions. Chen and Xiao (2015) consider how the supply chain’s efficiency is affected by various possible channel power when there are both kinds of uncertainties.

Another related stream of literature examines the topic of value of information. Raju and Roy (2000) consider the scenario where competing firms forecast demand using market information-gathering techniques. They find that the firm can increase its profit as the forecast becomes more precise, and a change in the forecast precision exerts a large influence on the firm’s profit when the uncertainty in demand is large. Hess and Lucas (2004) answer the question of how much marketing research (to better understand demand) should be performed that uses production resources otherwise. Gal-Or et al. (2008) study whether or
not a manufacturer should share information about demand with its retailers when different firms have different degrees of accuracy concerning their demand signals. They assert that the manufacturer may share information with the less informed retailer only if transmitting information is costly. Christen et al. (2009) investigate whether the manufacturer should learn a lot about a narrow range of markets or learn a little about a lot of markets when investment is scarce. They posit that focusing resources on a few markets is optimal when processing information is costly, or when the firm has accurate prior knowledge about the unknown parameters and is efficient in processing information. Pun and Heese (2015) study a firm’s optimal advertising and market research investments when the marketing budget is limited. They find that investment in advertising can be small when stockout is costly, and investment in market research can be small when the retail price is large. However, the papers mentioned in this paragraph consider a single type of uncertainty only; as such, they do not directly compare supply uncertainty and demand uncertainty.

In the literature on simulation and statistics, studies have been performed on variation reduction methods. Wu et al. (2006) consider the optimal decision for a risk-averse newsboy, using the mean-variance approach. Yang et al. (2007) use the active-set method and the Newton search procedure to solve the case of a buyer who needs to make a supplier selection decision in the presence of random demand. However, unlike our paper, these studies do not consider the possibility that the supply chain can exert effort to reduce uncertainty. Hong (2009) express the quantile sensitivities as conditional expectations. As a result, they use an infinitesimal perturbation analysis estimator and another more consistent estimator using the conditional expectation form. Glasserman et al. (2000a) present an algorithm for estimating the value at risk using Monte Carlo simulation and 'delta-gamma approximation. Glasserman et al. (2000b) use transform inversion and numerical approximation to calculate value at risk when the risk factors have a multivariate t-distribution, and Hoogerheide and van Dijk (2010) use a sampling method called the “Quick Evaluation of Risk using Mixture of tt approximations” to calculate the expected shortfall and
the value at risk measures in a Bayesian updating framework. However, these authors’ methods, contents and contexts are different than the ones presented in this paper.

To the best of our knowledge, our paper is the first to conduct an uncertainty reduction analysis that considers both demand and supply uncertainty and compares the impacts of both uncertainties. Further, it is the first to show how uncertainty reduction efforts for both uncertainties can change a firm’s profit.

3. Without Uncertainty-Reduction Effort

In order to achieve a clear understanding of the two uncertainties, in this section, we first focus on the case where there is no option of exerting effort to reduce uncertainties. We will extend our consideration to the case where the firm can exert efforts to reduce uncertainties in Section 4. Furthermore, we discuss each uncertainty separately in Sections 3 and 4, and define the following two scenarios: under scenario S, we consider the case where supply is uncertain but demand is deterministic; under scenario D, we consider the case where supply is deterministic but demand is uncertain. We then demonstrate the robustness of our results by considering the case where both uncertainties exist simultaneously in Subsection 4.3. Table 1 lists all the notations that are used in this paper.

3.1. Model

Consider a centralized system where there is a central planner who is responsible for both supply and demand. Without loss of generality, the expected demand is normalized to 1. The unit benefit is $r$ and the unit shortage (goodwill) cost is $g \geq 0$. The production cost is $c$ per unit. The output of the production is imperfect (e.g., because the raw material is imperfect), and the expected yield rate is $y$. We assume that $r \geq c/y \geq 0$. Since uncertainty may exist in either supply or demand, we consider two scenarios with different sources of uncertainty below.

Under scenario S, the demand is deterministic but the supply is uncertain. We use standard deviation to measure uncertainty, with the motivation that reduction in variation
Table 1 Table of notations

<table>
<thead>
<tr>
<th>parameters</th>
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<tbody>
<tr>
<td>( r )</td>
<td>unit benefit</td>
</tr>
<tr>
<td>( g )</td>
<td>unit shortage cost</td>
</tr>
<tr>
<td>( y )</td>
<td>expected yield rate</td>
</tr>
<tr>
<td>( c )</td>
<td>unit production cost</td>
</tr>
<tr>
<td>( Z_S ) or ( Z_D )</td>
<td>uncertainty, with cdf ( F )</td>
</tr>
<tr>
<td>( \sigma_S ) or ( \sigma_D )</td>
<td>standard deviation of ( Z_S ) or ( Z_D )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>calculated value</th>
<th></th>
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<tbody>
<tr>
<td>( k )</td>
<td>critical fractile</td>
</tr>
<tr>
<td>( \overline{TC}_S ) or ( \overline{TC}_D )</td>
<td>total cost when there is no effort</td>
</tr>
<tr>
<td>( TC_S, TC_D ) or ( TC_{SD} )</td>
<td>total cost when there is effort</td>
</tr>
<tr>
<td>( \Delta TC_S ) or ( \Delta TC_D )</td>
<td>improvement in total cost</td>
</tr>
<tr>
<td>( F )</td>
<td>cumulative distribution function of uncertainty</td>
</tr>
<tr>
<td>( f )</td>
<td>probability density function of uncertainty</td>
</tr>
<tr>
<td>( G(z) )</td>
<td>( \int_0^z y f(y) dy )</td>
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<table>
<thead>
<tr>
<th>decision variables</th>
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<tbody>
<tr>
<td>( x_S ) or ( x_D )</td>
<td>order quantity</td>
</tr>
<tr>
<td>( e_S ) or ( e_D )</td>
<td>effort to reduce uncertainty</td>
</tr>
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</table>

(standard deviation) means fewer shortages.\(^1\) As is commonly seen in the literature (e.g., Chick et al. 2008, Yang et al. 2009, 2012), we use yield-rate uncertainty as a proxy for the supply uncertainty. We model the yield rate with \( yZ_S \), where \( Z_S > 0 \) is a random variable. We assume that \( Z_S \) belongs to the location-scale family distribution, represented by a cdf \( F \), with a mean of one and variance \( \sigma_S^2 \). Then the yield rate \( yZ_S \) has a mean of \( y \) and variance of \( y^2\sigma_S^2 \). The location-scale family distribution is also commonly used in the literature (e.g., Zhang 2005, Yan and Zhao 2011). Examples of distribution that belong to the location-scale family are uniform, triangular, gamma, beta and Weibull distributions.

The firm must decide the number of production units \( x_S \) in order to minimize the total cost:

\[
E[\overline{TC}_S] = x_S c + gE[1 - x_S y Z_S] + rE[\min(1, x_S y Z_S)].
\]

Next, consider scenario D, where the supply is deterministic, but the demand is uncertain, with value \( Z_D > 0 \). We assume that \( Z_D \) also belongs to the location-scale family and has a mean of one, variance of \( \sigma_D^2 \) and cdf of \( F \). The firm must then decide on the number of production units \( x_D \) in order to minimize the expected total cost:

\(^1\) We can see this outcome in the extreme case where the standard deviation is zero (i.e., there is no uncertainty). In this case the model is completely deterministic, and hence, there is no shortage.
\[ E[TC_D] = x_Dc + gE[Z_D - x_Dy]^+ - rE[\min(Z_D, x_Dy)]. \] (2)

3.2. Analysis

Since supply uncertainty and demand uncertainty are unrelated, the random variables \( Z_S \) and \( Z_D \) are independent. Moreover, in order to compare both uncertainties on the same scale, we assume that the random variables \( Z_S \) and \( Z_D \) are independent and identically distributed.

Recall that the critical fractile of the NV model is as follows:

\[ k \equiv \frac{g + r - c/y}{g + r}. \] (3)

The overage cost is \( c/y \) and the underage cost is \( g + r - c/y \). And define \( G(z) \equiv \int_0^z yf(y)dy \). Proposition 1 compares the number of production units under both scenarios. For discussion purposes, we illustrate Propositions 1 and 2 in Figure 1, assuming that the uncertainties under both scenarios are uniformly distributed. (Note that we use uniform distributions in Figure 1 just for exposition convenience; the shape of the curve in Figure 1 is robust across different distributions.)

**Proposition 1:** The optimal production quantities, \( x_S \) and \( x_D \), satisfy the following:

a. \( x_S = \frac{1/y}{G^{-1}(1-k)} \) and \( x_D = \frac{1}{y}F^{-1}(k) \).

b. \( x_S > x_D \Leftrightarrow F^{-1}(k)G^{-1}(1-k) < 1 \).

The two types of uncertainties have different structural impacts, and therefore, the number of production units under the two uncertainties is different as well. This difference stems from the fact that the number of production units has a distinct impact on the two types of uncertainty: under scenario \( D \), the number of production units is a supply decision and is independent of demand, so it does not influence the demand uncertainty; under scenario \( S \), the variance of the random supply depends on the supply decision, so the number of production units influences the supply uncertainty of the final product.
Regardless of the nature of the uncertainty (supply uncertainty versus demand uncertainty), the firm’s optimal production quantity is small (large) when the critical fractile is small (large). The comparison of the two production quantities depends on the expression $F^{-1}(k)G^{-1}(1-k)$. When the critical fractile is small, it can be shown that this expression is small; therefore, supply uncertainty results in a larger production quantity. On the other hand, when the critical fractile is intermediate, this expression is large, so demand uncertainty would result in a larger production quantity.

Proposition 2 compares the total cost under both scenarios, which allows us to determine which type of uncertainty is more costly.

**Proposition 2**: The expected total costs, $E[TC_S]$ and $E[TC_D]$, satisfy the following:

a. $E[TC_S] = -r + (g + r)F[G^{-1}(1-k)]$ and $E[TC_D] = g - (g + r)G[F^{-1}(k)]$.

b. $E[TC_S] > E[TC_D] \iff F[G^{-1}(1-k)] + G[F^{-1}(k)] > 1$.

Under scenario S, supply uncertainty is costly when the underage cost is sufficiently smaller than the overage cost. This is because the production quantity $x_S$ is large when the production cost is small (Proposition 1), so the firm can mitigate uncertainty by producing...
a large quantity. However, both types of uncertainties are not costly when the production cost is small because the firm can produce a large production quantity to mitigate the uncertainty (Proposition 1).

When the critical fractile is small, supply uncertainty causes the firm to produce more units, and we observe that this response causes supply uncertainty to be less costly. In Section 4, we will see that the ability to exert uncertainty-reduction efforts has a non-trivial impact on the results in Propositions 1 and 2.

4. With Uncertainty-Reduction Effort

In this section, we consider the case where supply and demand uncertainties can be reduced by exerting efforts (e.g., obtaining more information by auditing the supplier or surveying potential customers), and we study the impact of such efforts on these two uncertainties.

4.1. Model

First, consider Scenario S, where supply is uncertain and demand is deterministic. The firm makes an effort $e_S$ to reduce the supply uncertainty $Z_S$ (i.e., $\frac{\partial \sigma_S}{\partial e_S} < 0$), and the effort has decreasing marginal effects, i.e., $\frac{\partial^2 \sigma_S}{\partial e_S^2} > 0$. Consistent with the related literature (e.g., Raju and Roy 2000, Gal-Or et al. 2008, Christen et al. 2009), we assume that the firm’s supply and demand estimations are unbiased, and hence, efforts do not change the expected value.

Then the total cost under Scenario S is:

$$E[TC_S] = e_S + x_S c + gE[1 - x_S y Z_S] + rE[\min(1, x_S y Z_S)].$$  (4)

Next, consider Scenario D, where the supply is deterministic, but the demand $Z_D$ is uncertain. The firm decides the level of effort $e_D$ to reduce the demand uncertainty $Z_D$ (i.e., $\frac{\partial \sigma_D}{\partial e_D} < 0$), and the effort has decreasing marginal effects, i.e., $\frac{\partial^2 \sigma_D}{\partial e_D^2} > 0$. Then the total cost under Scenario D is as follows.

$$E[TC_D] = e_D + x_D c + gE[Z_D - x_D y] + rE[\min(Z_D, x_D y)].$$  (5)
4.2. Analysis

To derive additional analytical insights, in this subsection, we assume that $Z_S$ and $Z_D$ are independent and uniformly distributed random variables; that is, $Z_S = U[1 - \sigma_S, 1 + \sigma_S]$ and $Z_D = U[1 - \sigma_D, 1 + \sigma_D]$, where $0 \leq \sigma_S, \sigma_D \leq 1$. Moreover, we assume that $e_S$ has the following impact on supply uncertainty $Z_S$: $\sigma_S = \frac{1}{1 + e_S}$. Without exerting any effort, supply has maximum uncertainty (i.e., $Z_S \sim U[0, 2]$). Effort investment reduces uncertainty (i.e., $\frac{\partial \sigma_S}{\partial e_S} < 0$), and the marginal effect of effort investment decreases (i.e., $\frac{\partial^2 \sigma_S}{\partial e_S^2} > 0$). When the level of effort is infinity ($e_S = \infty$), all uncertainty is removed (i.e., $Z_S = 1$). Similarly, we assume that the demand uncertainty reduction effort has the following impact on demand uncertainty $Z_D$: $\sigma_D = \frac{1}{1 + e_D}$.

Proposition 3 compares the expected profitability of reducing the supply uncertainty and of reducing the demand uncertainty.

**Proposition 3:** There exists a unique $\hat{\sigma}$ such that $\frac{\partial E[TC_S]}{\partial \sigma_S}|_{\sigma_S = \sigma} > \frac{\partial E[TC_D]}{\partial \sigma_D}|_{\sigma_D = \sigma} \iff \sigma < \hat{\sigma}$.

The profitability for reducing uncertainty is large when a unit decrease in uncertainty leads to a large reduction in total cost. Then Proposition 3 states that when the uncertainty is sufficiently small, reducing supply uncertainty is more profitable than reducing demand uncertainty. Furthermore, it can be shown that $\hat{\sigma}$ increases in the critical fractile. As a result, reducing supply uncertainty is more profitable than reducing demand uncertainty when the financial consequences of uncertainty are large, when unit production cost is small or when the uncertainty is small.

Proposition 4 compares the optimal efforts under the two scenarios.

**Proposition 4:**

a. $e^*_S = \frac{g+r-2}{g+r} \sqrt{\frac{c}{y} \left( \frac{g+r-c/y}{g+r-1} \right)} - \frac{2c/y}{g+r}$ and $e^*_D = \sqrt{\frac{c}{y} \left( \frac{g+r-c/y}{g+r-1} \right)} - 1$.

b. There exists a unique $\hat{c}$ such that $e^*_S > e^*_D \iff c < \hat{c}$.

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2 All structural insights hold even when adding a parameter to capture the difference in cost effectiveness between demand uncertainty reduction effort and supply uncertainty reduction effort, i.e., $\sigma_D = \frac{1}{1 + \beta e_D}$. However, for ease of exposition, we present the result when this parameter is set to one, i.e., $\beta = 1$. 
The optimal efforts exerted under the two scenarios are very different because the firm uses different means to reduce different uncertainties. Specifically, under scenario \( D \), the firm can exert effort to influence the demand uncertainty only. In contrast, under scenario \( S \), not only can the firm exert effort to directly influence supply uncertainty, but it can also use production size to indirectly influence supply uncertainty. This difference - between the “one-channel” impact on uncertainty under scenario \( D \) and the “two-channel” impact on uncertainty under scenario \( S \) - leads to the firm’s different incentives in exerting effort to reduce uncertainty.

The comparisons of the two scenarios are based on well-behaved functions of \( g + r \) and \( c/y \) only. Therefore, we can characterize the entire solution on two-dimensional graphs. We consider the region where both efforts are positive.\(^3\) Figure 2 presents the comparison of efforts under the two scenarios; the graph on the left shows which effort is larger than the other one and the graph on the right details this comparison with values of ratio \( e^*_S/e^*_D \). In both graphs, the white region represents the insensible region where effort is not positive. We find that when the unit production cost is low, the firm should exert more effort in reducing supply uncertainty since doing so is more profitable than reducing demand uncertainty when the unit production cost is small, so the effort into reducing supply uncertainty becomes large.

From the optimal efforts, the optimal number of production units and the total cost under both scenarios can be obtained as follows:

\[
x^*_S = \frac{1}{y} \sqrt{\left( \frac{e^*_S}{e^*_S + 1} \right)^2 + \frac{4c/y}{(g+r)(e^*_S + 1)}}.
\]

\[
x^*_D = \frac{1}{y} \left( 1 + \frac{g + r - 2c/y}{\sqrt{c/y(g + r)(g + r - c/y)}} \right),
\]

\[
E[TC^*_S] = \frac{g + r}{2} \left( \sqrt{(e^*_S)^2 + \frac{4c/y(e^*_S + 1)}{g + r}} - e^*_S \right) + e^*_S - r \quad \text{and}
\]

\(^3\) It can be shown that \( e^*_S > 0 \Leftrightarrow g + r - c/y > \frac{4(g+r-1)}{g+r} \) and \( e^*_D > 0 \Leftrightarrow \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{(g+r)}} < \frac{g+r-c/y}{g+r} < \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{g+r}}. \)
Moreover, we define the improvement in total cost because of the option of uncertainty reduction efforts as follows:

\[ \Delta TC_S = E[TC_S(e_S = 0)] - E[TC_S^*] \]
\[ \Delta TC_D = E[TC_D(e_D = 0)] - E[TC_D^*]. \]  

Recall from Proposition 1 that in the absence of efforts, when the critical fractile is small, supply uncertainty would result in a larger production quantity than would demand uncertainty. When there is an option to exert efforts, we find that demand uncertainty would always result in a larger production quantity (\( x_S^* < x_D^* \)).

We plot the comparison of improvement in total costs and optimal total costs under the two scenarios in Figure 3 and Figure 4. The interpretation of these figures is similar to Figure 2.

We find that the option of supply uncertainty reduction effort would have a larger improvement in total cost when the production cost is small. This is because when the production cost is small, reducing the supply uncertainty is more profitable compared to
reducing the demand uncertainty, and therefore, the optimal effort is larger (Proposition 4).
However, since supply uncertainty itself is much more costly than demand uncertainty, the expected total cost is still larger, even after exerting efforts.

4.3. Simultaneous Uncertainties and with Uncertainty-Reduction Effort
We considered the two uncertainties separately in Subsections 4.1 and 4.2. In this subsection, we illustrate the robustness of our results by examining the extended scenario wherein two uncertainties exist simultaneously, which is denoted as Scenario SD. Similar to the single-uncertainty scenarios, under scenario SD, the yield rate is given by $yZ_S$ and the
demand is denoted by $Z_D$. $Z_S$ and $Z_D$ are independent random variables that are uniformly distributed, that is, $Z_S = U[1 - \sigma_S, 1 + \sigma_S]$ and $Z_D = U[1 - \sigma_D, 1 + \sigma_D]$. To deal with the shortage due to the uncertainties, the firm can exert both efforts, $e_S$ and $e_D$, to reduce the uncertainties in supply and in demand, respectively. Efforts have the following impacts on the uncertainties: $\sigma_S = \frac{1}{1 + e_S}$ and $\sigma_D = \frac{1}{1 + e_D}$.

Then the total cost under Scenario SD is:

$$E[TC_{SD}] = e_S + e_D + x_{SD}c + gE[Z_D - x_{SD}yZ_S] + rE[\min(Z_D, x_{SD}yZ_S)].$$

(12)

The analysis under scenario SD is more complex than that under the two single-uncertainty scenarios for two reasons: (1) Each uncertainty has a different impact on the equilibrium solution, so the efficiency of effort exerted to reduce uncertainty is also different, and (2) the two uncertainties are related to each other since they both affect the system cost. Through our analysis, we aim to provide insights that explain how the two uncertainties affect the total cost and how to create the optimal dual-effort plan to reduce both uncertainties.

We demonstrate the observations through a numerical study due to the intractability of the analysis in scenario SD. We assume that $y = 0.6$. Note that the choice of $y$ does not affect the numerical results. This is because from equation (12), $y$ always appears in the group with $x_{SD}$, i.e., $x_{SD}y$. Therefore, any change in $y$ only leads to an according proportional change in the first-best solution of $x_{SD}$ but does not influence the total system profit and the decisions on the uncertainty reduction efforts. Moreover, the terms $g$ and $r$ always appear as a group in the form of $g + r$, which we call as the net benefit of product. We set a range for $g + r$, i.e., $g + r \in [10, 15]$. Note that with other choices of the range of $g + r$, the numerical results do not be significantly influenced and the observations and the insights obtained from the numerical study still hold. Finally, we consider the cases where the last cost parameter, unit production cost $c$, takes the values such that the overage cost $c/y = 3, 4, 5$, and 6. We do not consider the cases where $c/y$ takes the values out of

4 Once again, all structural insights hold even when there is a difference in cost effectiveness between demand uncertainty reduction effort and supply uncertainty reduction effort, i.e, $\sigma_S = \frac{1}{1 + \beta_S e_S}$ and $\sigma_D = \frac{1}{1 + \beta_D e_D}$. For ease of exposition, we present the result when $\beta_S = \beta_D = 1$. 

the range $[3, 6]$ because those values lead to trivial cases of the first-best effort decisions. Specifically, when $c/y < 3$ (e.g., $c/y = 2$) or $c/y > 6$ (e.g., $c/y = 7$), the first-best solution $e^c_D = 0$ for most cases for $g + r \in [10, 15]$.

We use an algorithm with nested loops to calculate the first-best solution $\{e^c_D, x^c_{SD}, e^c_S\}$ in the centralized system. There are two challenges that make the numerical study difficult: first, the two uncertainties intermingle with each other in the system cost function $E[TC_{SD}]$. For example, each of the two terms $gE[Z_D - x_{SD}yZ_S]$ and $rE[\min(Z_D, x_{SD}yZ_S)]$ depends on the distributions of two random variables $Z_D$ and $Z_S$, which makes it challenging to solve the first-best solution $\{e^c_D, x^c_{SD}, e^c_S\}$; second, even under the assumption of uniform distributions of $Z_D$ and $Z_S$, we find that the optimal production effort decision must satisfy a cubic equation, which makes it hard to obtain the first-best solution $x^c_{SD}$.

Recall that $\sigma_S = \frac{1}{1+\epsilon_S}$ and $\sigma_D = \frac{1}{1+\epsilon_D}$. To see the above two challenges more clearly, we rewrite the system cost $E[TC_{SD}]$ as follows,

$$E[TC_{SD}] = \frac{1-\sigma_S}{\sigma_S} + \frac{1-\sigma_D}{\sigma_D} + y + x_{SD}c - (g+r)E[\min(Z_D, x_{SD}yZ_S)].$$  \hspace{1cm} (13)

We already know $Z_D \in [1-\sigma_D, 1+\sigma_D]$ and $x_{SD}yZ_S \in [x_{SD}y(1-\sigma_S), x_{SD}y(1+\sigma_S)]$. Define $A = 1-\sigma_D$, $B = 1+\sigma_D$, $a = x_{SD}y(1-\sigma_S)$, and $b = x_{SD}y(1+\sigma_S)$. To calculate $E[\min(Z_D, x_{SD}yZ_S)]$, we need to compare the values of $A, B, a,$ and $b$. In Table 2, we provide the system profit $E[TC_{SD}]$ as a function of the production effort $x_{SD}$ in differences cases of $\{A, B, a, b\}$.

Note that we only need to consider the four cases of $\{A, B, a, b\}$ in Table 2, since in all other cases the system cost turns out to be trivial linear function of the production effort $x_{SD}$ and hence the first-best solution of $x_{SD}$ cannot be in those other cases. In Table 2, we see that in cases $a \leq A \leq b \leq B$ and $A \leq a \leq B \leq b$, the first-best solution of $x_{SD}$ must be solved from a cubic equation. This makes it very difficult to solve the closed-form solution of the first-best quantity $x^e_{SD}$, and makes it even more difficult to solve the closed-form solution of the first-best effort decisions $(e^c_S, e^c_D)$ based on $x^c_{SD}$. 
Fortunately, by overcoming these challenges, we obtain sufficient numerical results to demonstrate meaningful insights. Figure 5 presents the optimal production quantity and Figure 6 demonstrates the optimal uncertainty-reduction efforts.

<table>
<thead>
<tr>
<th>${A, B, a, b}$</th>
<th>$E[T_c]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \leq A \leq b \leq B$</td>
<td>$\frac{1}{2\sigma_p} \left( \frac{(1 + \sigma_d)^2 y^2 x_{SD}^2}{6\sigma_s} + \frac{(1 - \sigma_d)^2 (1 - \sigma_p)}{4\sigma_s} y x_{SD} - \frac{(1 + \sigma_d)(1 - \sigma_p)^2}{4\sigma_s} + \frac{(1 - \sigma_d)^3}{12\sigma_s y x_{SD}} \right)$</td>
</tr>
<tr>
<td>$a \leq A \leq b \leq B$</td>
<td>$\frac{1 - \sigma_d^2 y x_{SD}}{4\sigma_s} + \frac{1 + \sigma_d}{2\sigma_s} - \frac{3 + \sigma_d}{12\sigma_s y x_{SD}}$</td>
</tr>
<tr>
<td>$A \leq a \leq B \leq b$</td>
<td>$\frac{1}{2\sigma_s} \left( \frac{(1 - \sigma_d)^2 y^2 x_{SD}^2}{12\sigma_d} - \frac{(1 - \sigma_d)^2 (1 + \sigma_d)}{4\sigma_P} y x_{SD} + \frac{1 + \sigma_d}{y x_{SD}} \left( \frac{(1 + \sigma_d)^2}{6\sigma_d} - \frac{(1 - \sigma_d)^2}{4\sigma_d} - 1 \right) \right)$</td>
</tr>
<tr>
<td>$A \leq a \leq b \leq B$</td>
<td>$\frac{(1 + \sigma_d) y x_{SD}}{2\sigma_d} - \frac{3 + \sigma_d^2 y^2 x_{SD}^2}{12\sigma_d} - \frac{(1 - \sigma_d)^2}{4\sigma_d}$</td>
</tr>
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Figure 5: Optimal production quantity

We obtain the following observations and insights from our numerical experiments.

a. Figure 5 demonstrates the first-best quantity for different values of the net benefit of product $(g + r)$ and overage cost $(c/y)$. Similar to the two single-uncertainty scenarios, the optimal production quantity increases when the net benefit of product $(g + r)$ rises. This is because the higher net benefit of product stimulates the firm to produce more product to
obtain a higher profit. In addition, when the overage cost ($c/y$) increases the production quantity gets smaller since the higher cost discourages the firm to produce more.

b. The optimal uncertainty-reduction efforts are depicted in Figure 6. First, we see that the firm has incentives to exert more efforts to reduce the uncertainties in either supply or demand when the net benefit of product is higher or the overage cost is lower. This is because when the net benefit of product is higher or the overage cost is lower, production quantity increases. Therefore, since the supply (demand) uncertainty reduction effort is exerted to reduce the mismatches between the quantity and supply (demand), when quantity increases, the uncertainty reduction efforts become more beneficial as they lead to less loss in system profit due to these mismatches.

c. From Figure 6 we see that as in the single-uncertainty scenarios (Proposition 4), the slope of the first-best supply uncertainty reduction effort $e^c_S$ curve is steeper than the slope of the first-best demand uncertainty reduction effort $e^c_D$ curve. This implies that when the net benefit of production becomes higher, i.e., the cost parameters in the supply chain
allow more efforts exerted to reduce uncertainty in supply and demand, it is always more beneficial for the firm to exert more additional supply uncertainty reduction effort than demand uncertainty reduction effort.

d. Figure 6 shows that it is possible for the firm to exert higher overall effort in either supply uncertainty reduction or demand uncertainty reduction. Specifically, when the net benefit of product is low or when the overage cost is high, the effort exerted to reduce demand uncertainty is higher than the effort exerted to reduce supply uncertainty. Otherwise, the demand uncertainty reduction effort is lower. This is because when the net benefit of product is low or when the overage cost is high, production quantity is low (Figure 5), reducing supply uncertainty becomes less profitable and so the first-best supply uncertainty reduction effort is lower than the first-best demand uncertainty reduction effort.

From our results in this subsection, we see that the findings and intuition from the single-uncertainty analysis hold when both uncertainties exist at the same time, and we conclude that our results are robust.

5. Conclusion

The uncertain natures of supply and demand stand out as the main drivers for the common shortages in many industries, such as health care, consumer goods and agriculture. In this paper, we consider the scenario where a firm exerts effort to reduce demand and supply uncertainties, and also decides the production quantity. We analyze the impact of exerting efforts to reduce supply and demand uncertainties on production quantity and total cost. When the financial consequences of shortages are significant, we find that (a) supply uncertainty is more costly than demand uncertainty, (b) supply uncertainty reduction effort is more profitable than demand uncertainty reduction effort, (c) after exerting efforts, supply uncertainty would always cause the firm to produce fewer units than would demand uncertainty, and (d) although the option of a supply uncertainty reduction
effort delivers a larger improvement to total cost, supply uncertainty still leads to a higher cost than does demand uncertainty.

Our results have managerial implications for a firm that faces both supply and demand uncertainty. The firm can first determine the relative cost of shortage compared to other costs. If shortage cost is relatively large, the firm should focus its efforts on reducing supply uncertainty first (e.g., establishing better communication with its suppliers, finding and working with more suppliers and improving production yield). On the other hand, if shortage cost is relatively small, the firm should exert effort to reduce demand uncertainty first (e.g., market research for its products, targeted consumer surveys and better demand forecasting). Second, the firm should remember that exerting supply uncertainty reduction effort always causes the firm to produce fewer units than exerting demand uncertainty reduction effort. Although supply uncertainty reduction effort delivers a larger degree of improvement to total cost (and hence, supply uncertainty reduction effort is more efficient), the reduced supply uncertainty still leads to a higher system cost than does the reduced demand uncertainty.

Next we consider our insights with the two companies that we had had interviews with (i.e., Intel and a large Korean consumer electronics firm). For a new (CPU) product launch of Intel, there is demand uncertainty as well as supply uncertainty (e.g., yield/production uncertainty). Furthermore, net benefit (and the shortage cost) is high. Hence we expect Intel to focus on lowering supply uncertainty to lower its costs at the first phases of a new product life cycle. As the product becomes mature, the focus shifts from supply uncertainty to demand uncertainty since net benefit decreases and overage cost increases. These are in line with our observations of the CPU production and market; production/yield uncertainty decreases over time, more CPUs are being manufactured and prices go down. The nature of business is different for the Korean company. The net benefit is low, volume is high, demand uncertainty is high, there is supply uncertainty and overage cost is high. Therefore, we expect the Korean firm to tackle demand uncertainty first and this is in line with the industry trends such as marketing, advertisement and stiff competition.
Our paper is based on a stylized model, and it has some limitations. We consider a single-period model, and uncertainty may change over time. It would therefore be interesting to examine a multi-period problem where having information on one period can minimize uncertainty in the subsequent periods. Future research may also seek to examine how competition in the end market or the component market affects the optimal solution. Despite these limitations, we believe that our approach will foster an understanding of and hedging against shortages, and we expect that the suggested approach will be useful for practice and research in this area.

References


Evans, P. 2015. Target closes all 133 stores in Canada, gets creditor protection. [Retrieved 2015-02-05].


**Appendix**

**Proof of Proposition 1:**

Part a: $E[\hat{TC}_S] = x_S c + (g + r) \int_0^{\frac{1}{\sigma^2}} (1 - x_S y) f(z) dz - r$. $E[\hat{TC}_S]$ is convex in $x_S$ because

$$\frac{\partial^2 E[\hat{TC}_S]}{\partial x_S^2} = \frac{g + r}{x_S y} f\left(\frac{1}{x_S y}\right) > 0.$$ 

By solving the FOC, we have $x_S = \frac{1}{y G'(1-k)}$ and $E[\hat{TC}_S] = -r + (g + r) F[G^{-1}(1-k)]$. Similarly, $E[\hat{TC}_D] = x_D c + (g + r) \int_{x_D y}^{\infty} (z - x_D y) f(z) dz - r$. \(\frac{\partial^2 E[\hat{TC}_D]}{\partial x_D^2}\) = $(g + r)y^2 f(x_D y) > 0$. By solving the FOC, we have $x_D = \frac{1}{y} F^{-1}(k)$. Moreover, $E[\hat{TC}_D] = g - (g + r) G[F^{-1}(k)]$ because $\int_{F^{-1}(k)}^{\infty} [z - F^{-1}(k)] f(z) dz = 1 - G[F^{-1}(k)] - (1-k) F^{-1}(k)$.

Part b: $x_S > x_D \iff \frac{1}{y G'(1-k)} > \frac{1}{y} F^{-1}(k) \iff F^{-1}(k) G^{-1}(1-k) < 1.$ $\square$

**Proof of Proposition 2:**

Part a: The results are from the proof of Proposition 1.

Part b: $E[\hat{TC}_S] > E[\hat{TC}_D] \iff -r + (g + r) F[G^{-1}(1-k)] > g - (g + r) G[F^{-1}(k)] \iff F[G^{-1}(1-k)] + G[F^{-1}(k)] > 1.$ $\square$

**Proof of Proposition 3:**

For uniform distribution $F(z) = \frac{z-1+\sigma}{2\sigma}$ and $G(z) = \frac{z^2 - (1-\sigma)^2}{4\sigma}$. Then we have $E[\hat{TC}_S]|_{\sigma_s = \sigma} = c_S - r + (g + r) \sqrt{\frac{(1+\sigma)^2 - 4\sigma k - 1 + \sigma}{2\sigma}}$ and $E[\hat{TC}_D]|_{\sigma_D = \sigma} = c_D + g - (g + r) k (1 - \sigma + \sigma k)$. Therefore,
擺列等式時，為了使質量和測量等式都呈正數，得到 

$$\sqrt{1+k-\sigma_0} > \sqrt{1+k} > 0$$

和

$$\sqrt{1+k} > 0, \quad \omega(\Omega > 0), \quad \omega = k(1-k)^2 - 2k(1-k)^2 - (1-k)^2 + 2(1-k).$$

因此，指數係數為 $k(1-k)$，是正的。更進一步，令 $\omega(\Omega = 0)$，當 $\Omega(\sigma = 0) < 0 \iff k < 0.5$。因此，當 $\sigma < 0$，我們定義 $\sigma = 0$。相似地，當 $\sigma > 1$，我們定義 $\sigma = 1$。然後結果就成立。

**Proof of Proposition 4:**

Part a: The result can be obtained by substitution using uniform distribution and how the efforts affect the standard deviation.

Part b: We consider the non-trivial region where both efforts are positive. $e_S^* > 0 \iff g + r - c/y > \frac{4(g+r-1)}{g+r}$. Moreover, $e_D^* > 0 \iff \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{g+r}} < \frac{g+r-c/y}{g+r} < \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{g+r}}$. It can be shown that $\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{g+r}} < \frac{4(g+r-1)}{(g+r)^2}$. Therefore, both efforts are positive (i.e., $e_S^* > 0$ and $e_D^* > 0$) if and only if $\frac{4(g+r-1)}{(g+r)^2}$.

$e_S^* > e_D^* \iff (g+r-2)\sqrt{k(1-k)} - (1-2k)\sqrt{g+r-1} > \sqrt{(g+r)(g+r-1)k(1-k)}$. It follows that both sides are positive when $\frac{4(g+r-1)}{(g+r)^2} < k < \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{g+r}}$. Then, squaring both sides and simplifying gives

$$(7(g+r)-8)k^2 - (7(g+r)-8)k + g + r - 1 > 2(g+r-2)(1-2k)\sqrt{k(1-k)(g+r-1)}$$

(14)

$$(7(g+r)-8)k^2 - (7(g+r)-8)k + g + r - 1 < 0 \iff k > 1/2 \quad \text{and} \quad 2(g+r-2)(1-2k)\sqrt{k(1-k)(g+r-1)} < 0$$

Region 1) $k > \frac{7(g+r)-8+\sqrt{21(g+r)^2-52(g+r)+32}}{2(7(g+r)-8)}$: (14) is true because the LHS of (14) is positive and the RHS of (14) is negative.
Region 2) \(1/2 < k < \frac{7(g+r)-8+\sqrt{21(g+r)^2-52(g+r)+32}}{2(7(g+r)-8)}\): Both sides of (14) are negative. Then, squaring both sides and simplifying gives \((g+r)(16(g + r)^2 - 31(g + r) + 16)k^4 - 2(g + r)(16(g + r)^2 - 31(g + r) + 16)k^3 + (g + r)(20(g + r)^2 - 37(g + r) + 18) - 2(g + r)(g + r - 1)(2(g + r) - 1)k + (g + r - 1)^2 < 0\). There are four roots to this equation, but only one root \(\hat{k} = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{8(g+r)^3-19(g+r)^2+12(g+r)-8(g+r-2)(g+r-1)\sqrt{(g+r)(g+r-1)}}{(g+r)(16(g+r)^2-31(g+r)+16)}}\) is in region 2.

Region 3) \(\frac{7(g+r)-8-\sqrt{21(g+r)^2-52(g+r)+32}}{2(7(g+r)-8)} < k < 1/2\): (14) is false because the LHS of (14) is negative and the RHS of (14) is positive.

Region 4) \(k < \frac{7(g+r)-8-\sqrt{21(g+r)^2-52(g+r)+32}}{2(7(g+r)-8)}\): Both sides of (14) are positive. Using a similar logic as that in Region 2, squaring both sides and simplifying shows that (14) is false. \(\square\)